SEMIEMPIRICAL SOLUTION OF HEAT-TRANSFER PROBLEMS FOR NUCLEATE BOILING

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On the basis of semiempirical turbulent transport theory, with the dynamic velocity expressed in terms of the energy dissipation factor controlling the transport process, equations are derived for the heat-transfer coefficient for the nucleate boiling of liquids in the cases of pool boiling and boiling in vertical pipes.

The heat-transfer process associated with the nucleate boiling of a liquid is realized as a result of both convection and the generation of vapor, which contributes to the formation and growth of vapor bubbles [1]. However, for moderate heat inputs and relatively low pressures, such that the density of vaporization centers is small, heat transfer takes place primarily due to convection. The heat resistance in this case is concentrated mainly in a wall layer of liquid, the thickness of which is determined by the turbulence state of its vapor bubbles formed on the heat-transfer surface. If we assume that the heat-transfer law for nucleate boiling is consistent with the laws of convective transport, we can analyze this process by the general methods used for the solution of such problems.

We propose to analyze the nucleate boiling process on the basis of the semiempirical theory of turbulent transport [2-4]. The heat-transfer laws for steady liquid flow over a heat-transfer surface are described with reasonable accuracy by means of this theory when the characteristics of the turbulent fluctuations in the flow can be expressed in terms of the tangential stresses τ_0 and when the dynamic velocity is given by the relation

$$u_* = \sqrt{\frac{\tau_0}{\rho}}.$$

It has been shown in a series of examples in [5-8] that if the dynamic velocity is expressed in terms of the transport-controlling energy dissipation E_0 (energy dissipation in the wall layer) in the form

$$u_* = \sqrt[4]{\frac{\nu E_0}{\rho}}, \tag{1}$$

the semiempirical theory can also be used to advantage for the solution of many other problems in heat transfer and hydrodynamics (heat transfer at the wall of a bubble tower or in equipment that uses mixers, heat transfer and drag in gas–liquid flows, etc.); the heat-transfer process in all these cases is described by the equation [9]

$$\frac{\alpha}{\lambda} \cdot \frac{\nu}{u_*} = \frac{\mathrm{Pr}}{\psi}. \tag{2}$$

Here $\psi = f(u_*, \mathrm{Pr})$; the form of this function is given in [6, 9] and elsehwere. The magnitude of the energy dissipation controlling the transport process for steady liquid flow over a plane heat-transfer surface and in pipes is equal to the energy dissipation in the laminar substrate ($E_0 = \tau_0^2/\rho\nu$) and can be calculated in the same way as the ratio of the total flow-dissipated power

$$N = \tau_0 s w \tag{3}$$

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to the volume of the wall layer

$$V = s\delta_r, \tag{4}$$

$$E_0 = \frac{N}{V}. \tag{5}$$

As we know, the relationship between the thicknesses of the hydrodynamic and thermal boundary layers is expressed as follows:

$$\delta_h = \delta_t \, \mathrm{Pr}^{1/3}.$$

The thickness of the thermal boundary layer (λ/α) can be represented as follows on the basis of (2):

$$\delta_t = \frac{\nu}{u_*} \cdot \frac{\psi}{\mathrm{Pr}},$$

and, since $\psi/\mathrm{Pr} \sim \mathrm{Pr}^{-1}/^3$, it follows that

$$\delta_h \sim \frac{\nu}{u_*}. \tag{6}$$

Taking (3), (4), and (6) into account, we find that Equation (5) takes the form

$$E_0 = \frac{N}{V} \sim \frac{\tau_0 w u_*}{\nu}.$$

After the substitution of this expression into (1) we arrive at the relation

$$u_*' \sim \sqrt[3]{\frac{\tau_0 w}{\rho}} \sim \sqrt[3]{\left(\frac{\tau_0}{\rho}\right)^{3/2}} = \sqrt{\frac{\tau_0}{\rho}}. \tag{7}$$

For the more general case in which the wall layer contains a uniformly distributed turbulence source of total power N we readily deduce the following relation from expressions (1), (4), (5), and (6) for the determination of u_*:

$$u_* \sim \sqrt[3]{\frac{N}{s\rho}}. \tag{8}$$

This relation can be used in conjunction with Eq. (2) to analyze the heat-transfer process in nucleate boiling if the vaporization centers are represented as uniformly distributed turbulence sources of power

$$N = \frac{Az'}{\tau_g}.$$

The number z' of vaporization centers acting simultaneously is related to the total number z of centers by the expression

$$z' = z \frac{\tau_g}{\tau_h},$$

and, since $\tau_h = 1/f$, we have

$$N = Azf. \tag{9}$$

With regard for Eq. (9), expression (8) assumes the form

$$u_*'' \sim \sqrt[3]{\frac{Afn}{\rho}}. \tag{10}$$

In order to calculate the work A done by one bubble in growing on the heat-transfer surface up to the instant of breakoff we use the following expression characterizing the growth of a vapor bubble [10]:

$$R = a\tau^{1/2},$$

in which

$$a = \sqrt{\frac{2\beta\lambda\Delta T}{r\rho_r}}, \tag{11}$$

where β is a coefficient depending only on the extreme contact angle. This equation implies that the instantaneous growth rate of the vapor bubble is

$$u = \frac{dR}{d\tau} = \frac{a^2}{2R}. \tag{12}$$

The drag force of the bubble surface in the liquid can be expressed approximately by the relation

$$F \sim \rho u^2 R^2$$

or, taking the dependence (12) into account,

$$F \sim \rho a^4. \tag{13}$$

After integration with regard for (13) the expression for the work $A = \int_{R_b}^{R_0} FdR$ acquires the form $A \sim \rho a^4 (R_0 - R_b)$, where R_6 is the radius of the bubble nucleus.

Inasmuch as $R_0 \gg R_b$, we can assume that

$$A \sim \rho a^4 R_0. \tag{14}$$

After the substitution of (14) into Eq. (10), the latter assumes the form

$$u_*'' \sim \sqrt[3]{a^4 R_0 f n}. \tag{15}$$

It has been verified experimentally in [11, 12] that the product $R_0 f$ = const over a wide range of variation of heat inputs and it can be calculated [13] according to the equation

$$u_0 = 2R_0 f = 0.59 \left(\frac{\sigma g \Delta\rho}{\rho^2} \right)^{1/4}. \tag{16}$$

Using expressions (11), (16), we readily reduce relation (15) to the form

$$u_*'' = k \left(\frac{\lambda}{r\rho_v} \right)^{2/3} u_0^{1/3} \Delta T^{2/3} n^{1/3},$$

and by substitution of u_*'' into Eq. (2) we obtain the following relation for α:

$$\alpha = k \frac{\lambda^{5/3}}{\nu} \cdot \frac{u_0^{1/3}}{(r\rho_v)^{2/3}} \cdot \frac{\mathrm{Pr}}{\psi} \Delta T^{2/3} n^{1/3}. \tag{17}$$

In order to check the validity of Eq. (17) it suffices to compare it with the published experimental and theoretical data on the influence of the bubble generation frequency f, density n of vaporization centers, and temperature differential ΔT on the heat-transfer coefficient.

It follows from Eq. (17) that for moderate heat inputs the heat-transfer coefficient is independent of f. This fact has been established experimentally in [14].

A simultaneous measurement of the heat-transfer coefficient and the number of active vaporization centers as a function of the time in protracted water boiling with q = const has been carried out in [15]. It follows from the experimental data of [15] for $\alpha \sim n^{1/5}$. An analogous relation is also obtained from (17) for q = const. In [12] the relation $\alpha \sim \Delta T^{2/3} n^{1/3}$ is given, which is completely consistent with (17); in [16-18] the power exponents differ, but only slightly.

We infer from the foregoing comparisons that the solution of the heat-transfer problem for nucleate pool boiling in the regime of dominant heat transfer is theoretically possible by means of the semiempirical turbulent transport theory, which yields good results. In order to determine the number of vaporization centers we use the following approximate relation given in [19]:

$$n = 6.25 \cdot 10^{-14} L \left(\frac{r\rho_v \Delta T}{T_s \sigma} \right)^3, \tag{18}$$

in which L = 1 m. Equation (17) with (18) taken into account gives the relation $\alpha = f(\Delta T)$ and correctly reflects the influence of the physical properties of the liquid and vapor on the heat-transfer coefficient.

For the analysis of boiling in the case of large vaporization center densities it is required, as in [1], to take into account the heat of formation of the bubbles growing on the heat-transfer surface.

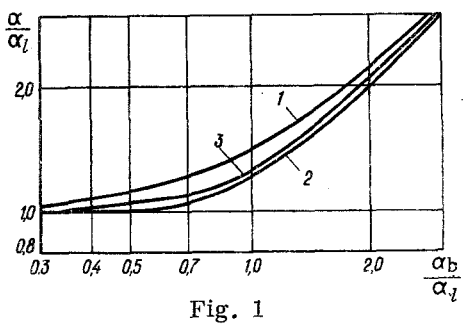

Fig. 1

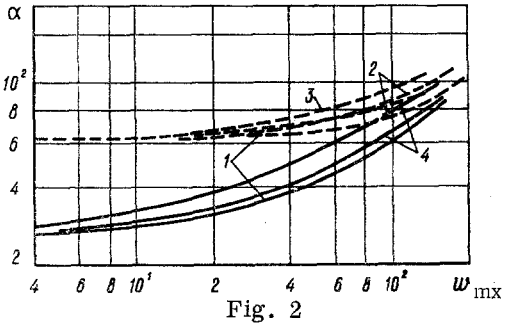

Fig. 2

Fig. 1. Ratio α/α_l versus α_b/α_l. 1) According to the Kutateladze equation [21]; Labuntsov equation [22]; 3) author's equation (21).

Fig. 2. Heat-transfer coefficient α, kW/m^2 deg, versus mixture flow velocity w_{mx}, m/sec, at a pressure of $1.9 \cdot 10^{-6}$ N/m^2. According to Eq. (21): 1) $w_l = 0.33$; 2) 1; 3) 2 m/sec; 4) according to the Borishanskii equation (22). For the dashed curve q $= 13.9 \cdot 10^5$ W/m^2; for the solid curve q $= 3.48 \cdot 10^5$ W/m^2.

The applicability of turbulent transport theory for the analysis of nucleate pool boiling also affords an approach to the solution of the heat-transfer problem for the boiling of directionally flowing liquids in pipes.

For the solution of this problem we introduce an additional assumption, namely that the heat-transfer process is characterized by the total power dissipated in the medium. The admissibility of this assumption has been demonstrated experimentally in the solution of the heat-transfer problem for gas–liquid flows in pipes [7]. The power of one of the sources of agitation of vapor bubbles growing on the heat-transfer surface is characterized by Eq. (9). Besides the latter, in an ascending flow of a vapor–liquid mixture the power needed to transport the liquid upward, except for that part used in the formation of a potential and kinetic energies, will be dissipated in the liquid. For a short section of pipe of height H it may be assumed that w_V = const, whereupon

$$N' = (w_v + w_l)\,\Delta p \cdot \frac{\pi d^2}{4} - w_l \rho g H \frac{\pi d^2}{4},$$

where $\Delta p = pgH(1-\varphi) + 4\tau_0 H/d$ is the pressure drop over the length H.

After suitable transformations we arrive at the expression

$$N' = \pi d H \tau_0 w_{mx} + w_{re}\varphi\,(1-\varphi)\,\rho g H,$$

in which

$$w_{re} = \frac{w_v}{\varphi} - \frac{w_l}{1-\varphi}.$$

The total power, on the other hand, characterizing the heat-transfer process in the given situation can be represented in the form

$$N = Azf + \pi d H w_{mx}\tau_0 + w_{re}\varphi\,(1-\varphi)\,\rho g H.$$

In the determination of the energy dissipation E_0 the first two terms must be referred to the wall layer volume $\pi d H \delta_g$, while the third term, which characterizes the power losses in the relative motion of the phases, must be referred [7, 8] to the total liquid volume in the pipe, $V = (\pi d^2/4)H(1-\varphi)$, because the relative motion elicits nearly isotropic turbulence. Taking into account expression (7) and the one-way permeation of fluctuations from the flow core to the wall layer [7], we obtain the following expression for E_0:

$$E_0 = k'\left(\frac{Afn}{\nu}u_* + \frac{\tau_0 w_{mx}}{\nu}u_*\right) + \varkappa^4 w_{re}\rho g\varphi\,(1-\varphi)^2$$

and a corresponding expression for the calculation of u_*, taking (1), (7), and (10) into account:

$$u_*^4 = u_*''^3 u_* + u_*'^3 u_* + u_*'''^4, \qquad (19)$$

where

$$u_*''' = \varkappa \left(vg\omega_{re}\varphi(1-\varphi)^2\right)^{1/4},$$

and $\varkappa = 1.9$ is a proportionality factor [7]. The determination of the exat solution of Eq. (19) is problematical. However, within 3% error limits u, can be found from the approximate relation

$$u_* = \left[(u_*'^3 + u_*''^3)^{4/3} + u_*'''^4\right]^{1/4}. \tag{20}$$

Recognizing that the dimensionless temperature ψ in Eq. (2) depends only very slightly on u_*, we transform Eqs. (20) and (2) to the following:

$$\alpha = \left[(\alpha_0^3 + \alpha_b^3)^{4/3} + \alpha_{re}^4\right]^{1/4}, \tag{21}$$

in which, with regard for relations (2) and (7),

$$\alpha_0 = \alpha_l \left(\frac{\tau_0}{\tau_l}\right)^{1/2}.$$

The quantity α_{re} has to be included only for small liquid flow velocities, small heat fluxes, and large volume contents of vapor, i.e., for $\alpha_{re} > \alpha_0$ and $\alpha_{re} > \alpha_b$. Note that in deriving Eqs. (20) and (21) it was assumed that the law $2R_0f = u_0$ determined experimentally for pool boiling carries over to the case of forced circulation of the liquid and that, in addition, the density of vaporization centers is independent of the liquid flow velocity. These assumptions, of course, require experimental corroboration. The investigation of [20] proves, however, that $\alpha_b \sim \alpha_{pb}$, i.e., for fully developed boiling in pipes the way in which various factors affect the heat-transfer rate is the same as in boiling under free-convection (pool) conditions. Moreover, a comparison of relation (21) with the equations of Kutateladze [21] and Labuntsov [22] (Fig. 1), which hold for small vapor contents, such that $\alpha_{re} \ll \alpha_0$, $\alpha_{re} \ll \alpha_b$, and $\tau_0 \approx \tau_l$, and with the equation of Borishanskii et al. [23]:

$$\frac{\alpha}{\alpha_{pb}} = 0.65\left[1 + 1.5\cdot 10^{-8}\left(\frac{w_{mx}r\rho}{q}\right)^3\right]^{1/2}, \tag{22}$$

which is valid over a wide range of vapor contents in the mixture (Fig. 2), implies that the foregoing assumptions are reliable. In the comparison of Eqs. (21) and (22) the ratio τ_0/τ_l was calculated from the Martinelli–Nelson relation [24], and it was assumed [20] that $\alpha_b = 0.65\alpha_{pb}$.

NOTATION

R	is the vapor bubble radius;
R_0	is the vapor bubble breakoff radius;
d	is the pipe diameter;
H	is the pipe length;
δ_g	is the thickness of hydrodynamic boundary layer;
δ_t	is the thickness of thermal boundary layer;
τ	is the time;
τ_g	is the bubble growth time;
τ_n	is the bubble nucleation time;
f	is the bubble formation frequency;
u_*	is the dynamic velocity; $u_0 = 2R_0f$;
u_0	is the bubble growth rate;
w	is the average flow velocity of liquid;
w_v, w_l	are the reduced velocities of vapor and liquid;
w_{re}	is the relative velocity of phases;
$x_{mx} = w_v + w_l$	is the velocity of vapor–liquid mixture;
g	is the free-fall acceleration;
s	is the surface area;
τ_0	is the tangential stress at wall;
τ_l	is the tangential stress at wall during liquid flow;
E_0	is the transport-controlling energy dissipation;
A	is the work done by one bubble during growth on a heat-transfer surface;
q	is the specific heat flux;
z	is the number of vaporization centers;

T_s is the saturation temperature;
ΔT is the temperature differential between wall and liquid;
φ is the true volumetric vapor content;
ρ is the liquid density;
ρ_v is the vapor density;
$\Delta\rho = \rho - \rho_v$;
ν is the kinematic viscosity of liquid;
σ is the surface tension;
λ is the thermal conductivity of liquid;
α is the heat transfer coefficient;
r is the latent heat of evaporation;
α_b is the boiling heat-transfer coefficient;
α_{pb} is the pool boiling heat-transfer coefficient;
α_l is the pipe-flow of the heat-transfer coefficient of liquid;
α_{re} is the heat-transfer coefficient for vapor bubbling of liquid;
ψ is the dimensionless temperature differential;
Pr is the Prandtl number;
$\varkappa$, k, k' is the proportionality factors;
$n = z/s$ is the density of vaporization centers.

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